Correspondence between the contracted BTZ solution of cosmological topological massive gravity and two-dimensional Galilean conformal algebra

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Abstract

We show that BTZ black hole solution of Cosmological Topological Massive Gravity (CTMG) have a hidden conformal symmetry. In this regard, we consider the wave equation of a massless scalar field propagating in BTZ spacetime and find the wave equation could be written in terms of the $SL(2,R)$ quadratic Casimir. From the conformal coordinates, the temperatures of the dual CFTs could be read directly. Moreover, we compute the microscopic entropy of the dual CFT by Cardy formula and find a perfect match to Bekenstein-Hawking entropy of BTZ black hole. Then we consider Glilean conformal algebras (GCA), which arises as a contraction of relativistic conformal algebras ($x \to \epsilon x, \quad t \to t, \ \epsilon \to 0$). We show that there is a correspondence between GCA_2 on the boundary and contracted BTZ in the bulk. For this purpose we obtain the central charges and temperatures of GCA_2. Then we compute the microscopic entropy of the GCA_2 by Cardy formula and find a perfect match to Bekenstein-Hawking entropy of BTZ black hole in non-relativistic limit. The absorption cross section of a near region scalar field also matches to microscopic absorption cross section of the dual GCA_2. So we find further evidence that show correspondence between contracted BTZ black hole and 2-dimensional Galilean conformal algebra.

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1 Introduction

Recently, there has been some interest in extending the AdS/CFT correspondence to non-relativistic field theories [1], [2]. The Kaluza-Klein type framework for non-relativistic symmetries, used in Refs. [1], [2], is basically identical to the one introduced in [3] (see also [4]). The study of a different non-relativistic limit was initiated in [5], where the non-relativistic conformal symmetry obtained by a parametric contraction of the relativistic conformal group. Galilean conformal algebra (GCA) arises as a contraction relativistic conformal algebras [5], [6], where in $3+1$ space-time dimensions the Galilean conformal group is a fifteen parameter group which contains the ten parameter Galilean subgroup. Infinite dimensional Galilean conformal group has been reported in [6], the generators of this group are : $L^n = -(n+1)t^n x_i \partial_i - t^{n+1}\partial_t$, $M_i^n = t^{n+1}\partial_i$ and $J_{ij}^n = -t^n(x_i\partial_j - x_j\partial_i)$ for an arbitrary integer n, where i and j are specified by the spatial directions. There is a finite dimensional subgroup of the infinite dimensional Galilean conformal group which generated by $(J_{ij}^0, L^{\pm1}, L^0, M_i^{\pm1}, M_i^0)$. These generators are obtained by contraction ($t \to t,\ x_i \to \epsilon x_i,\ \epsilon \to 0,\ v_i \sim \epsilon$) of the relativistic conformal generators. Recently the authors of [7] (see also [8]) have shown that the GCA_2 is the asymptotic symmetry of CTMG in the non-relativistic limit. They have obtained the central charges of GCA_2, and also a non-relativistic generalization of Cardy formula. In the present paper we obtain similar result by another method. But our aim in this paper is more than this. We show that the BTZ black hole solution of CTMG have a hidden conformal symmetry, not only in the relativistic case, but also in the non-relativistic case. We show that for massless scalar field, in non-relativistic CTMG there exist a finite Galilean conformal symmetry acting on the solution space. According to our knowledge this is the first paper that study the hidden conformal symmetry for a black hole in the non-relativistic case.

Recent investigation on the holographic dual descriptions for the black holes have achieved substantial success. According to the Kerr/CFT correspondence [9], the microscopic entropy of four-dimensional extremal Kerr black hole was calculated by studying the dual chiral conformal field theory associated with the diffeomorphisms of near horizon geometry of Kerr black hole. Subsequently, this work was extended to the case of near-extreme black holes [10]. The main progress are made essentially on the extremal and near extremal limits in which the black hole near horizon geometries consist a certain AdS structure and the central charges of dual CFT can be obtained

by analyzing the asymptotic symmetry following the method in [11] or by calculating the boundary stress tensor of the $2D$ effective action [12]. Recently, Castro, Maloney and Strominger [13] have given evidence that the physics of Kerr black holes might be captured by a conformal field theory. The authors have discussed that the existence of conformal invariance in a near horizon geometry is not necessary condition, instead the existence of a local conformal invariance in the solution space of the wave equation for the propagating field is sufficient to ensure a dual CFT description(see also [14]). The scalar Laplacian in the low frequency limit could be written as the $SL(2,R)$ quadratic Casimir, showing a hidden $SL(2,R) \times SL(2,R)$ symmetries. In the microscopic description, using the Cardy formula for the microscopic degeneracy, they reobtain the Bekenestein-Hawking entropy of the black hole.

In the present paper we investigate the massless scalar wave equation in the background of BTZ black hole solution of CTMG and show the wave equation can be written in terms of $SL(2,R)$ Casimir invariants. From the conformal coordinates introduced in [9], we read the temperature of the dual CFT. The microscopic counting support this holographic picture. Then we consider the non-relativistic limit of both side of this correspondence, i.e, the non-relativistic limit of $2D$ CFT which give us GCA_2 from one side, and non-relativistic limit of CTMG which give us contracted BTZ from another side. For this purpose we obtain the central charges and temperatures of GCA_2. Then we showe that the radial part of Klein-Gordon (KG) equation in the background of contracted BTZ black hole, where ($j \to \epsilon j$, $\varphi \to \epsilon\varphi$), can be given by the non-realtivistic limit of quadratic Casimir of $SL(2,R)$. We could read the GCA_2 temperatures from the correspondence of radial part of non-relativistic KG equation and Casimir of GCA_2. Then we compute the microscopic entropy of the GCA_2 by Cardy formula and find a perfect match to Bekenstein-Hawking entropy of contracted BTZ black hole. After that in section 4, we compute the absorption cross section of a near region scalar field and find perfect match to the microscopic cross section in dual GCA_2. These results supports the idea of a correspondence between contracted BTZ black hole and dual Galilean conformal algebra in $2-$dimension.

2 Massless Scalar field in the background of CTMG

In this section we introduced the idea of the hidden conformal symmetry into the CTMG, and obtain the Virasoro algebras as a local symmetry of massless

scalar fields propagating in the BTZ black hole background. The existence of the 2-dim CFT behind the asymptotically AdS_3 spacetime, including the BTZ black hole solutions, was already pointed out by using the Brown-Henneaux's method, see [11] for the case without the gravitational Chern-Simons term, and [15] for the case with the Chern- Simons term.
We show that for massless scalar field Φ propagating in the background of cosmological topological massive gravity (CTMG), there exist a $SL(2,R)_L \times SL(2,R)_R$ conformal symmetry acting on the solution space. The BTZ space-time is given by the line element [7]

$$ds^2 = (-f(r) + \frac{16G^2 j^2}{r^2})dt^2 + \frac{dr^2}{f(r)} + r^2 d\varphi^2 + 8Gjdtd\varphi \qquad (1)$$

where

$$f(r) = (\frac{r^2}{l^2} - 8GM + \frac{16G^2 j^2}{r^2}) \qquad (2)$$

which is solution of the Einstein equation. The event horizons of the space-time are given by the singularities of the metric function which are the real roots of $r^2 f(r) = 0$. In the above metric G, j, M and $-\frac{2}{l^2}$ are gravitational constant, rotational parameter, mass of black hole and cosmological constant respectively. But their definitions in CTMG are

$$M = m + \frac{1}{\mu}\frac{j}{l^2} \qquad J = j + \frac{1}{\mu}m \qquad (3)$$

where $\frac{1}{\mu}$ is coupling constant of gravitational Chern-Simons term. Now we consider a bulk massless scalar field Φ propagating in the background of (1). The Klein- Gordon(KG) equation

$$\Box\Phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\Phi = 0 \qquad (4)$$

can be simplified by assuming the following from of the scalar field

$$\Phi(t, r, \theta, \varphi) = \exp(-im\varphi + i\omega t)S(\theta)R(r) \qquad (5)$$

and for $l = 1$ is reduced to the following equation

$$[\partial_u(\Delta\partial_u) + \frac{1}{4}[\frac{(\omega r_+ - mr_-)^2}{(u-u_+)(u_+ - u_-)} - \frac{(\omega r_- - mr_+)^2}{(u-u_-)(u_+ - u_-)}]]R(u) = 0 \qquad (6)$$

where

$$\Delta = uf(u) \qquad u = r^2 \qquad u_\pm = r_\pm^2 \qquad (7)$$

and

$$r_{\pm} = \sqrt{2G(m+j)} \pm \sqrt{2G(m-j)} \tag{8}$$

We can show that this equation can be reproduced by the introduction of conformal coordinates. We introduce the conformal coordinates [9].

$$\omega^{+} = \sqrt{\frac{u-u_{+}}{u-u_{-}}} \exp(2\pi T_R \varphi + 2n_R t) \tag{9}$$

$$\omega^{-} = \sqrt{\frac{u-u_{+}}{u-u_{-}}} \exp(2\pi T_L \varphi + 2n_L t) \tag{10}$$

$$y = \sqrt{\frac{u_{+}-u_{-}}{u-u_{-}}} \exp(\pi(T_R + T_L)\varphi + (n_R + n_l)t) \tag{11}$$

We define left and right moving vectors by

$$H_1 = \partial_{+} \tag{12}$$

$$H_0 = (\omega^{+}\partial_{+} + \frac{1}{2} y \partial_y), \tag{13}$$

$$H_{-1} = ((\omega^{+})^2 \partial_{+} + \omega^{+} y \partial_y - y^2 \partial_{-}) \tag{14}$$

$$\overline{H}_1 = \partial_{-} \tag{15}$$

$$\overline{H}_0 = (\omega^{-}\partial_{-} + \frac{1}{2} y \partial_y) \tag{16}$$

$$\overline{H}_{-1} = ((\omega^{-})^2 \partial_{-} + \omega^{-} y \partial_y - y^2 \partial_{+}) \tag{17}$$

which each satisfy the $SL(2,R)$ algebra

$$[H_0, H_{\pm 1}] = \mp H_{\pm 1}, \qquad [H_{-1}, H_1] = -2H_0 \tag{18}$$

and

$$[\overline{H}_0, \overline{H}_{\pm 1}] = \mp \overline{H}_{\pm 1}, \qquad [\overline{H}_{-1}, \overline{H}_1] = -2\overline{H}_0 \tag{19}$$

The quadratic Casimir is

$$H^2 = \widetilde{H}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1}H_1) = \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_- \quad (20)$$

The crucial observation is that these Casimir, when written in terms of φ , t, u

$$H^2 = \partial_u(\Delta\partial_u) - \frac{u_+ - u_-}{u - u_+}(\frac{T_L + T_R}{4A}\partial_t - \frac{n_L + n_R}{4\pi A}\partial_\varphi)^2 \quad (21)$$
$$+\frac{u_+ - u_-}{u - u_-}(\frac{T_L - T_R}{4A}\partial_t - \frac{n_L - n_R}{4\pi A}\partial_\varphi)^2 \qquad A = T_L n_R - T_R n_L$$

reduces to the radial equation (6), with these identifications

$$T_L = \frac{r_+ + r_-}{2\pi} \qquad T_R = \frac{r_+ - r_-}{2\pi} \quad (22)$$
$$n_L = \frac{r_+ + r_-}{2} \qquad n_R = \frac{r_- - r_+}{2}$$

The microscopic entropy of the dual CFT can be computed by the Cardy formula which matches with the black hole Bekenstein-Hawking entropy

$$S_{CFT} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) \quad (23)$$

The central charges of CTMG are

$$c_L = \frac{3}{2G}(1 + \frac{1}{\mu}) \qquad c_R = \frac{3}{2G}(1 - \frac{1}{\mu}) \quad (24)$$

From the central charges (24) and temperature (22) we have

$$S_{CFT} = \frac{\pi r_+}{2G} + \frac{1}{\mu}\frac{\pi r_-}{2G} \quad (25)$$

which agrees precisely with the gravity result. The contribution to the entropy that is due to the gravitational Chern-Simons (last term in eq.(25)) was first obtained by Solodukhin [16]. It is curiously proportional to the area of the inner horizon rather that of the outer horizon.

3 Galilean Conformal Algebra in 2-Dimension

Galilean Conformal Algebra in 2-dimensions can be obtained from contracting conformal algebra in 2-dimensions [6]. In two dimensions the non-trivial generators are given by

$$L_n = -(n+1)t^n x\partial_x - t^{n+1}\partial_t, \qquad M_n = t^{n+1}\partial_x \quad (26)$$

The above generators obey the following commutation relations , where define the Galilean conformal algebras.

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} \\
[L_m, M_n] &= (m-n)M_{m+n} \\
[M_n, M_m] &= 0
\end{aligned}
\tag{27}
$$

$2d$ Conformal algebra at the quantum level are described by two copy of Virasoro algebra.

$$
\begin{aligned}
[\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n} + \frac{c_R}{12}m(m^2-1)\delta_{m+n,0} \\
[\overline{\mathcal{L}}_m, \overline{\mathcal{L}}_n] &= (m-n)\overline{\mathcal{L}}_{m+n} + \frac{c_L}{12}m(m^2-1)\delta_{m+n,0}
\end{aligned}
\tag{28}
$$

From these, one obtains centrally extended 2d GCA

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + C_1 m(m^2-1)\delta_{m+n,0} \\
[L_m, M_n] &= (m-n)M_{m+n} + C_2 m(m^2-1)\delta_{m+n,0} \\
[M_n, M_m] &= 0
\end{aligned}
\tag{29}
$$

GCA central charges C_1 and C_2 are defined in terms of CFT central charges [7],

$$
C_1 = \lim_{\epsilon\to 0}\frac{c_L + c_R}{12} = \frac{1}{4G} \qquad C_2 = \lim_{\epsilon\to 0}(\epsilon\frac{c_L - c_R}{12}) = \frac{1}{4G\mu}
\tag{30}
$$

CFT entropy (23) with the limit ($\epsilon \to 0$) is converted to Galilean conformal entropy,

$$
S_{GCA} = \lim_{\epsilon\to 0}(\frac{\pi^2}{3}[6C_1(T_L + T_R) + 6C_2(\frac{T_L - T_R}{\epsilon})])
\tag{31}
$$

We define Galilean conformal temperatures as following

$$
T_1 = \lim_{\epsilon\to 0} 6(T_L + T_R) \qquad T_2 = \lim_{\epsilon\to 0} 6\frac{T_L - T_R}{\epsilon}
\tag{32}
$$

the GCA entropy is

$$
S_{GCA} = \frac{\pi^2}{3}(C_1 T_1 + C_2 T_2)
\tag{33}
$$

To make the GCA entropy finite, $T_L + T_R \sim O(1)$ and $T_L - T_R \sim O(\epsilon)$, as a result $r_+ \sim O(1)$ and $r_- \sim O(\epsilon)$. These results appear in the limits

of $j \to \epsilon j$ and $m \to m$, that correspond with the result of [7]. Finally from Eq.(22) and above discussion we have introduced finite n_1 and n_2 (in non-relativistic limit) in terms of n_L and n_R

$$n_1 = \frac{n_L + n_R}{\epsilon} \qquad\qquad n_2 = n_L - n_R \tag{34}$$

4 Massless Scalar field in the background of non-relativistic CTMG

In this section, we will show that for massless scalar field Φ, in non-relativistic CTMG there exist a finite Galilean conformal symmetry acting on the solution space. We consider a bulk massless scalar field Φ propagating in the background of (1). The KG equation (6) in non-relativistic limit ($j \to \epsilon j$, $\varphi \to \epsilon\varphi$) reduce to

$$\begin{aligned}
&\partial_u(\Delta\partial_u) - \frac{1}{4}[\frac{r_+'^2}{(u-u_+)(u-u_-)}\partial_t^2 \\
&\quad -2r_+'r_-'(\frac{1}{(u-u_+)(u-u_-)} - \frac{1}{(u-u_-)(u_+-u_-)})\partial_t\partial_\varphi]\Phi = 0 \\
&\partial_\varphi^2\Phi = 0
\end{aligned} \tag{35}$$

where $r_+' = 2\sqrt{2Gm}$ and $r_-' = \sqrt{\frac{2G}{m}}j$. We can show that the above equation can be reproduced by the conformal coordinate (9), (10), (11) in non-relativistic limit. H^2 is Casimir of $SL(2,R)_L \times SL(2,R)_R$ so

$$[H^2, H_{0\pm1}] = 0 \qquad\qquad [H^2, \overline{H}_{0\pm1}] = 0 \tag{36}$$

Since, $2D$ Galilean conformal generators are created by the mixing of $2D$ conformal generators in non-relativistic limit, from Eq.(36) it can be shown that Casimir of conformal group is the same Casimir of Galilean conformal group in non-relativistic limit. In another term, since

$$H'^2 = \lim_{\epsilon\to0} H^2, \qquad L_{0\pm1} = \lim_{\epsilon\to0}(H_{0\pm1} + \overline{H}_{0\pm1}), \qquad M_{0\pm1} = \lim_{\epsilon\to0}(\frac{H_{0\pm1} + \overline{H}_{0\pm1}}{\epsilon}) \tag{37}$$

we have

$$\begin{aligned}
&\lim_{\epsilon\to0}[H^2, H_{0\pm1} + \overline{H}_{0\pm1}] = 0 \qquad\qquad \lim_{\epsilon\to0}[H^2, \frac{H_{0\pm1} + \overline{H}_{0\pm1}}{\epsilon}] = 0 \\
&[H'^2, L_{0\pm1}] = 0 \qquad\qquad [H'^2, M_{0\pm1}] = 0.
\end{aligned} \tag{38}$$

So, the Casimir operator (21) in non-relativistic limit is Casimir of GCA (H'^2 is non-relativistic limit of H^2). Finally Casimir operator of GCA is

$$H'^2 = \partial_u(\Delta\partial_u) - \frac{u_+ - u_-}{u - u_-}(\frac{T_1}{A'^2})^2\partial_t^2 \tag{39}$$
$$-\frac{2(u_+ - u_-)}{\pi A'^2}(\frac{T_1 n_1}{u - u_+} - \frac{T_2 n_2}{u - u_-})\partial_t\partial_\varphi$$
$$+(\frac{1}{\epsilon^2}\frac{u_+ - u_-}{r - r_-}\frac{n_2^2}{\pi^2 A'^2} - \frac{u_+ - u_-}{r - r_+}\frac{n_1^2}{\pi^2 A'^2})\partial_\varphi^2 \qquad A' = T_1 n_2 - T_2 n_1$$

The above equation is reduced to the radial equation (35), with these identifications.

$$T_1 = \frac{6r'_+}{\pi} \qquad T_2 = \frac{6r'_-}{\pi} \qquad n_1 = r'_- \qquad n_2 = r'_+ \tag{40}$$

The microscopic entropy of the dual GCA can be computed by the non-relativistic Cardy formula (33). From the central charges (30) and temperatures (40) we have

$$S_{GCA} = \pi(\sqrt{\frac{2m}{G}} + \frac{j}{\mu}\sqrt{\frac{1}{2Gm}}) = S_{BH} \tag{41}$$

which agrees precisely with the gravity result, (in non-relativistic limit) presented in [7]. As we have mentioned in the introduction the authors of [7] have studied the Galilean non-relativistic limit of the dual field theory of BTZ black hole in CTMG. From the Galilean limit of Virasoro algebra and the Galilean limit of the BTZ black hole, they get the result that the entropy of the Galilean limit of BTZ black hole is consistent with the entropy calculated using Cardy formula from the Galilean limit of the Virasoro algebra. This agreement between our result and the result of [7] is interesting, and show that the non-relativistic version of the BTZ solution of CTMG really has hidden conformal symmetry in the near region of the black hole. In the next section by obtaining the absorption cross section we present final evidence of the existence of this hidden symmetry.

5 Absorption cross section

In this section, we give a brief review for scattering of the scalar field Φ which propagates in the (contracted) BTZ background [17]. We calculate the absorption cross section, from gravity side and matches the result with

2D (GCA) CFT cross section. Two-point function of conformal invariant fields is given by [10, 18]

$$G(t^+,t^-) \sim (-1)^{h_R+h_L}(\frac{\pi T_L}{\sinh(\pi T_L t^+)})^{2h_L}(\frac{\pi T_R}{\sinh(\pi T_R t^-)})^{2h_R} \tag{42}$$

where $t^{\pm}$ are the coordinates of the 2D CFT and (h_R, h_L) are eigenvalues of $\mathcal{L}_0$ and $\overline{\mathcal{L}}_0$ respectively. Absorption cross section in term of frequency and temperature, from Fermi's golden rule [10, 18] can be read

$$P_{abs} \sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+}[G(t^+ - i\delta, t^- - i\delta) - G(t^+ + i\delta, t^- + i\delta)] \tag{43}$$

Using the following integral

$$\int dx e^{(-i\omega x)}(-1)^{\Delta}(\frac{\pi T}{\sinh[\pi T(x \pm i\delta)]})^{2\Delta} = \frac{(2\pi T)^{2\Delta-1}}{\Gamma(2\Delta)} e^{\pm\omega/2T} \mid \Gamma(\Delta + i\frac{\omega}{2\pi T}) \mid^2 \tag{44}$$

the absorption cross section becomes

$$P_{abs} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}) \mid \Gamma(h_L + i\frac{\omega_L}{2\pi T_L}) \mid^2 \mid \Gamma(h_R + i\frac{\omega_R}{2\pi T_L}) \mid^2 \tag{45}$$

From above method, non-relativistic limit of absrption cross section can be computed. Non-relativistic limit of two-point function is given by

$$\lim_{\epsilon\to 0} G \sim (-1)^{\Delta}(\frac{T_1}{12\sinh(\frac{\pi T_1}{12}(t \pm i\delta))})^{2\Delta} \exp(\frac{2T_2}{T_1}\xi) \tag{46}$$

where scaling dimension $\Delta = h_L + h_R$ is eigenvalue of L_0 and rapidity $\xi = \lim_{\epsilon\to 0}[\epsilon(h_L - h_R)]$ is eigenvalue of M_0. From Eqs.(43), (44), (46) and using this relation

$$\lim_{x\to k}(1 + f(x))^{g(x)} = \lim_{x\to k}\exp(f(x)g(x)) \tag{47}$$

where

$$\lim_{x\to k} f(x) = 0, \qquad \lim_{x\to k} g(x) = \infty \tag{48}$$

the cross section for 2D GCA is given by

$$P_{abs} \sim \exp(\frac{2T_2}{T_1}\xi) T_1^{2\Delta-1} \sinh(\frac{6\omega_1}{T_1}) \mid \Gamma(\Delta + i\frac{6\omega_1}{\pi T_1}) \mid^2 \tag{49}$$

where $\omega_1 = \omega_L + \omega_R$. We can study, the absorption cross section of BTZ (contracted BTZ), from gravity side, in relativistic (non-relativistic) limit. The results can be verified in agreement with the corresponding results for the operator dual to the scalar field in the 2D CFT (45) (2D GCA (49)). The radial part of KG equation

$$(\Box - M^2)\Phi = 0 \tag{50}$$

for massive scalar field Φ, is given by

$$[\partial_u(\Delta\partial_u) + (\frac{A}{u-u_+} + \frac{B}{u-u_-} + C)]R(u) = 0 \tag{51}$$

where

$$A = \frac{(\omega r_+ - m r_-)^2}{4(u_+ - u_-)} \qquad B = -\frac{(\omega r_- - m r_+)^2}{4(u_+ - u_-)} \qquad C = \frac{M^2}{4} \tag{52}$$

The wave function satisfying the ingoing boundary condition at the horizon is of the form

$$R(z) = z^\alpha (1-Z)^\beta F(a, b, c; z) \tag{53}$$

where $z = \frac{u-u_+}{u-u_-}$, and

$$\alpha = \sqrt{A} \qquad \beta = \frac{1}{2}(1 - \sqrt{1-4C}) \qquad \gamma = \sqrt{-B} \tag{54}$$

where

$$c = 1 - 2i\alpha \qquad a = \beta + i(\gamma - \alpha) \qquad b = \beta - i(\gamma + \alpha) \tag{55}$$

The asymptotic form can be read out by taking the limit $z \to 1$ and $1 - z \to u^{-1}$

$$R(u) \sim D r^{2h-2} + E r^{-2h} \tag{56}$$

where

$$D = \frac{\Gamma(c)\Gamma(2h-1)}{\Gamma(a)\Gamma(b)} \qquad E = \frac{\Gamma(c)\Gamma(1-2h)}{\Gamma(c-a)\Gamma(c-b)} \tag{57}$$

and h is the conformal weight

$$h = \frac{1}{2}(1 + \sqrt{1-4C}) \tag{58}$$

Absorption cross section is captured by the coefficient D,

$$P_{abs} \sim \mid D \mid^{-2} \sim \sinh(2\pi\alpha) \mid \Gamma(a) \mid^2 \mid \Gamma(b) \mid^2 \tag{59}$$

To see explicitly that P_{abc} matches with microscopic greybody factor of the dual CFT we need to identify the conjugate charges, δE_L and δE_R defined by

$$\delta S_{BH} = \delta S_{CFT} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R} \tag{60}$$

we have

$$\delta E_L = \omega_L \qquad\qquad \delta E_R = \omega_R \tag{61}$$

where

$$\omega_R = \frac{\omega + m}{2(r_+ + r_-)} \qquad\qquad \omega_L = \frac{\omega - m}{2(r_+ - r_-)} \tag{62}$$

Hence, the coefficient a, b can be expressed in terms of parameters ω_L and ω_R

$$a = h_R + i\frac{\omega_R}{2\pi T_R} \qquad\qquad b = h_L + i\frac{\omega_L}{2\pi T_L} \tag{63}$$

where for the scalar field $h_L = h_R = h$. Similarly we have

$$2\pi\alpha = \frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R} \tag{64}$$

Finally from Eqs.(59, 63, 64), the absorption cross section can be expressed as

$$P_{abs} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}) \mid \Gamma(h_L + i\frac{\omega_L}{2\pi T_L}) \mid^2 \mid \Gamma(h_R + i\frac{\omega_R}{2\pi T_L}) \mid^2$$

which is the finite temperature absorption cross section for 2D CFT. In following, we consider the scattering of non-relativistic limit of BTZ, from gravity side, and matches the result with 2D GCA cross section (49). The radial part of KG equation in non-relativistic limit can be expressed in term of Eq.(51), with these identifications

$$A_n = \frac{(\omega r'_+ - m r'_-)^2}{4u'_+} \qquad\qquad B_n = -\frac{(\omega\epsilon r'_- - m r'_+/\epsilon)^2}{4u'_+} \qquad C = \frac{M^2}{4}$$

From Eqs.(54, 55), we see that

$$\lim_{\epsilon\to 0} a = a_n = \infty, \qquad \lim_{\epsilon\to 0} b = b_n = \infty, \qquad \lim_{\epsilon\to 0}(a+b) = finite \tag{65}$$

so coefficient D in non-relativistic limit is given by

$$D_n = \frac{\Gamma(c_n)\Gamma(2h-1)}{\Gamma(a_n+b_n)} \qquad \lim_{a\to\infty}\Gamma(a)\lim_{b\to\infty}\Gamma(b) \simeq \Gamma(a_n+b_n) \tag{66}$$

Absorption cross section from gravity side in the non-relativistic limit, is captured by coefficient D_n

$$P_{abs} \sim \mid D_n \mid^{-2} \sim \sinh(2\pi\alpha_n) \mid \Gamma(a_n+b_n) \mid^2 \tag{67}$$

To see explicitly that P_{abc} matches with microscopic greybody factor of the dual GCA we need to identify the conjugate charge, δE_1 defined by

$$\lim_{\epsilon\to 0}\delta S_{BH} = \delta S_{GCA} = \frac{\delta E_1}{T_1} \tag{68}$$

we have

$$\delta E_1 = \omega_1 = \lim_{\epsilon\to 0}(\omega_L + \omega_R) \tag{69}$$

where

$$\omega_1 = \frac{\omega r'_+ - m r'_-}{2r'_+} \tag{70}$$

Hence, the coefficients a_n, b_n can be expressed in term of parameters ω_L and ω_R

$$a_n + b_n = \Delta + i\frac{6\omega_1}{\pi T_1} \tag{71}$$

where for the scalar field $\Delta = 2h$. Similarly we have

$$2\pi\alpha_n = \frac{6\omega_1}{T_1} \tag{72}$$

Finally from Eqs.(67), (71), (72), the absorption cross section can be expressed as ($\xi = \epsilon(h_L - h_R) = 0$)

$$P_{abs} \sim T_1^{2\Delta-1}\sinh(\frac{6\omega_1}{T_1}) \mid \Gamma(\Delta + i\frac{6\omega_1}{\pi T_1}) \mid^2 \tag{73}$$

which is the absorption cross section for 2D GCA.

6 Conclusions

In this paper at first we showed that there exist a holographic $2D$ CFT description for BTZ black hole solution of CTMG. The key ingredient for this is the hidden conformal symmetry in the black hole. We considered the wave equation of a massless scalar field propagating in CTMG spacetime and found the wave equation could be written in terms of the $SL(2, R)$ quadratic Casimir. We read the temperatures of the dual CFT from conformal coordinates. We recovered the macroscopic entropy from the microscopic counting. Then we extended this study to the non-relativistic limit, and showed that there exist a correspondence between GCA_2 on the boundary and contracted BTZ in the bulk. By definition Galilean conformal temperatures as equation (32), we could obtain the finite GCA_2 entropy by equation (33). After that we showed that the radial part of Klein-Gordon (KG) equation in the background of contracted BTZ black hole, where ($j \to \epsilon j$, $\varphi \to \epsilon\varphi$), can be given by the non-realtivistic limit of Casimir H^2. We could read the GCA_2 temperatures T_1, T_2 from the correspondence of radial part of non-relativistic KG equation and GCA Casimir H'^2. The temperatures T_1, T_2 given by equation (40) are exactly equal with previous formula we have obtain in (32). Then we calculated the entropy of contracted BTZ black hole by using GCA Cardy formula and non-relativistic temperatures. Finally we have shown that the absorption cross section of a near-region scalar field match precisely to that of microscopic dual GCA side.

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